Smoothed Particle Hydrodynamics (SPH) for Modelling Microfluidic Flow through 3D Printed Copper Filters

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Abstract: This review article presents key aspects of applying Smoothed Particle Hydrodynamics (SPH) to model fluid filtration through FDM 3D-printed copper-based filters (80% Cu, 20% PLA composite filament) with microporosity. Conservation laws in continuum dynamics form the foundation of meshless SPH modeling. The complexity of modeling 3D printed copper-based filters lies in capturing the irregular shapes of microporosity. We present an initial SPH model that provides velocity and pressure distributions through a round pore, demonstrating better accuracy than the FEM model. Such SPH approach can accurately capture water flow parameters through complex porous structures. Unlike FEM, which relies on predefined finite elements, SPH is a Lagrangian method based on the distribution of virtual particles and periodic boundary conditions, making it well-suited for modelling irregular geometries. Combined with 3D printing, SPH can support the design of optimal structures for water filtration.

Keywords: Water filter, Micropores, Cu/PLA filament, Momentum conservation, Modelling

1. Introduction

Smoothed Particle Hydrodynamics (SPH) has shown good results in fluid-solid interactions, multiphase flows, and porous media [1], [2]. This paper presents key aspects of a physics-based SPH model along with an initial simulation of water flow through an ideal pore focusing on copper-based filters made via FDM 3D printing.

2. Fundamentals of Smoothed Particle Hydrodynamics (SPH)

SPH uses a finite number of collocation points in a finite-dimensional space, typically approximated by polynomial functions in numerical solutions [1]. It is based on the Lagrangian formulation, meaning that physical variables are represented by material points or particles that move through different locations in the volumetric space [2]. In SPH formulations, particles represent small pieces of the observed material continuum at various scales, including micro, meso and macro scales. The state of the whole system is determined by the sum of particles' states regarding their mass, position, velocity, acceleration, energy and other relevant physical properties. Particles in SPH closely resemble physical particles due to the requirement of permanent mass conservation.

In SPH, conservation laws expressed as partial differential equations (PDEs) are transformed into integrals using interpolation functions that provide so called "kernel estimates" of some physical properties for certain points in time and space. This is usually done for the discrete number of points, thus not requiring meshes since solutions of functions are calculated using values of physical quantities in discrete points and interpolation kernel. This kernel approximation is the basic element of the SPH method [2]. Another important SPH parameter is the scalar *h*, also called the dilatation parameter or smoothing length, analogous to the characteristic length in FEM. Kernel approximation transforms PDEs of physical variables into partial derivatives of the kernel function, which must have continuous derivatives matching the PDE's order, typically achieved using third- or fifth-order polynomials. Another key property of SPH is particle approximation, where the continuous representation is transformed into a discrete sum over all particle states within the observed domain. However, clusterization of particles can lead to instabilities and incorrect modelling results in some cases. SPH is based on the time evolution of density and can be used to simulate fluid flow at low Reynolds (Re) numbers [3], such as in the case of household water filters. SPH uses the laws of mass conservation (equation 1 for density evolution), linear momentum conservation (equation 2), and energy conservation (equation 3). The velocity evolution of particle *i* in SPH is given by equation (4) in case of quasi-incompressible fluid [3].

$$\frac{d\rho^i}{dt} = \rho^i \sum_{j \in NHD} \frac{m^i}{\rho^j} \left(v_\alpha^i - v_\alpha^j \right) \frac{\partial W^{ij}}{\partial x_\alpha^i} \tag{1}$$

$$\frac{dv_\alpha^i}{dt} = \sum_{j \in NHD} m^j \left(\frac{\sigma_{\alpha\beta}^j}{\rho^{j2}} + \frac{\sigma_{\alpha\beta}^i}{\rho^{i2}} \right) \frac{\partial W^{ij}}{\partial x_\beta^i} \tag{2}$$

$$\frac{dE^i}{dt} = -\frac{\sigma_{\alpha\beta}^i}{\rho^i} \sum_{j \in NHD} m^j \left(v_\alpha^j - v_\alpha^i \right) \frac{\partial W^{ij}}{\partial x_\beta^i} \tag{3}$$

$$\frac{dv^i}{dt} = \sum_{j \in NHD} m^j \left(\frac{p^j}{\rho^{j2}} + \frac{p^i}{\rho^{i2}} \right) \nabla_i W^{ij} + \sum_{j \in NHD} \frac{m^i \left(\mu^i + \mu^j \right) v^{ij}}{\rho^i \rho^j} \left(\frac{1}{r^{ij}} \frac{\partial W^{ij}}{\partial r^i} \right) + F^i \tag{4}$$

3. 3D printed copper-based filters

FDM 3D printing can fabricate porous structures for water filters. Porous structures made of copper (Cu) and polylactic acid (PLA) composite (80% Cu, 20% PLA composite filament used in FDM) is shown in Figure 1 [4]. Complex shape of microporosity can be seen what is challenging for modelling that uses finite elements. Meshless SPH method is suitable for modelling of fluid flow through those complex structures [2].
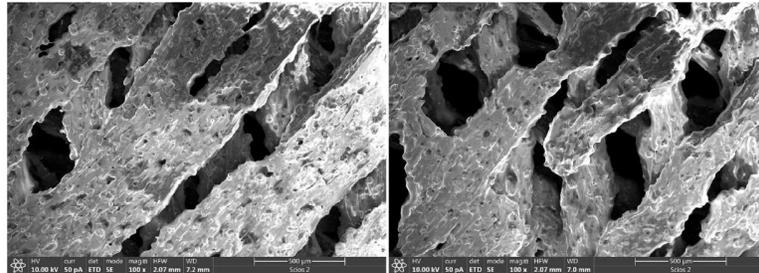

Figure 1. SEM image of porous structure fabricated by FDM 3D printing (reproduced from [4] under the terms and conditions of the Creative Commons Attribution (CC BY) license)

4. Modelling fluid flow through microporous media with SPH

SPH model with velocity and pressure distributions through one ideally round pore, with periodic boundary conditions in X and Y directions is shown in Figure 2.
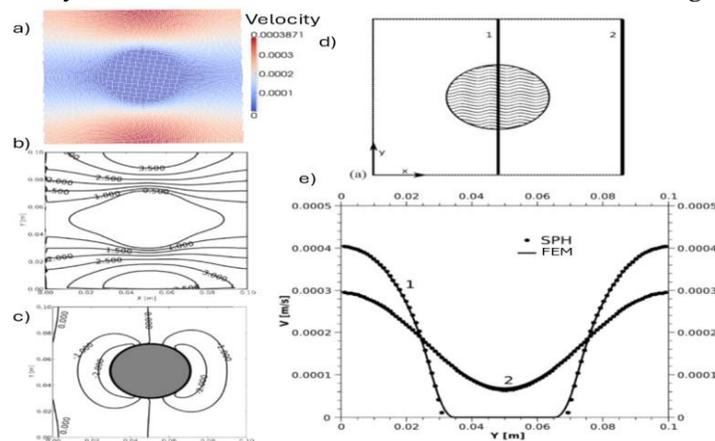

Figure 2. SPH model of fluid flow through one pore in porous water filter positioned within the pipe: a) particles distribution and absolute values of velocity, b) contour representation of velocities intensities; c) contour representation of pressure fields; d) positions of trajectories 1 and 2 in SPH model; e) comparison of velocities intensities over trajectories 1 and 2 obtained by using SPH and FEM approach.

Comparison between SPH and FEM models indicates better accuracy of SPH model. This approach to simulation of fluid flow can be specifically focusing on filtration, i.e., the capture, transport, and potential clogging of particles in a porous metallic medium, to simulate fluid flow carrying suspended particles (e.g., dust, microbes, colloids) through a 3D-printed filter, and study flow behaviour through micropores, particle

transport, capture, and clogging dynamics, including filtration efficiency, pressure drop, and permeability degradation over time.

5. Conclusions

This paper presented the Smoothed Particle Hydrodynamics (SPH) method for modelling microfluidic flow through 3D printed copper filters. The SPH model of single-pore flow showed excellent accuracy. Future work will address flow through irregular micropores, including particle transport and clogging from suspended pollutants.

Acknowledgment

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