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Recursive Identification of Takagi-Sugeno Models in the Presence of Piecewise Polynomial Disturbances

V. S. Đorđević + and V. Ž. Filipović ++

+ Department of Automatic Control, Robotics and Fluid Technique, University of Kragujevac, Faculty of Mechanical and Civil Engineering, Dositejeva 19, 36000 Kraljevo, Serbia, Phone: (381) 036-383-269, E-Mail: djordjevic.v@mfkv.kg.ac.rs

++ Department of Automatic Control, Robotics and Fluid Technique, University of Kragujevac, Faculty of Mechanical and Civil Engineering, Dositejeva 19, 36000 Kraljevo, Serbia, Phone: (381) 036-383-269, E-Mail: v.filipovic@mts.rs

***Abstract* – The paper considers recursive identification of Takagi-Sugeno (TS) models. The TS models consist of finite collection of linear time invariant systems. It is supposed that real systems can be described with the Hammerstein model for which the reasonable approximation is TS model. The disturbance is piecewise polynomial. As identification algorithm the Kaczmarz algorithm is used. Special attention is paid for design of input signal for Hammerstein system and verification signal. Cluster analysis is based on Gustafson-Kessel algorithm. From this analysis it follows determination of membership functions. As a result, the recursive algorithm has structure similar to instrumental variable method. Simulations cover the practical behaviour of algorithm.**

***Key words:* Takagi-Sugeno model, Kaczmarz algorithm, Gustafson-Kessel algorithm, piecewise polynomial disturbance**

I. INTRODUCTION

Fuzzy models well approximate nonlinear systems. This particularly refers to TS models where it is used linearization of nonlinear systems in fuzzy regions of the state space [1]. Input space is decomposed into a finite collection of fuzzy regions. The consequent function describes system behavioural in those regions.

In this paper, it is considered modelling of Hammerstein model in forward line of the system, shown on Fig. 1.

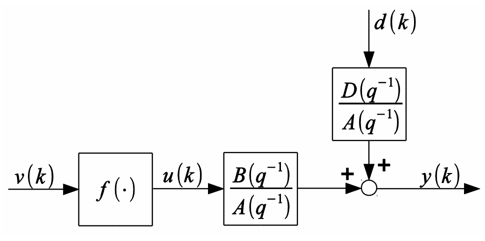

Fig. 1. The system with Hammerstein model.

A piecewise polynomial disturbance, with its own dynamics, acts on the system, and its form is shown in Fig. 2.

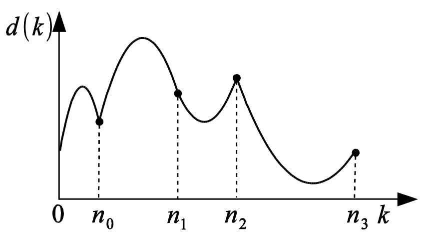

Fig. 2. An example of the piecewise polynomial disturbance.

The nature of this disturbance is more deterministic than stochastic. Piecewise polynomial disturbance is defined as follows

$$d_k = \begin{cases} f_0(k) = f_{00} + f_{01}k + \ldots + f_{0m_0}k^{m_0} & 0 \le k < n_0 \\ f_1(k) = f_{10} + f_{11}k + \ldots + f_{1m_1}k^{m_1} & n_0 \le k < n_1 \\ \vdots & \vdots \\ f_l(k) = f_{l0} + f_{l1}k + \ldots + f_{lm_l}k^{m_l} & n_{l-1} \le k < n_l \end{cases}, \quad (1)$$

and

$$\bar{m} = \max\{m_0, m_1, \ldots, m_l, \ldots\}. \quad (2)$$

During identification, signals which excite the system have an important role. Excitation signal must be such chosen that excite the largest possible range of amplitudes and frequencies. The selection of excitation signal quite depends on the system which is identified. In fuzzy models identification, signals, such as PRBS or Gaussian noise, do not give good results. On the other side, a multi-sinusoidal excitation signal, with the addition of a weak Gaussian noise, in most cases gives good results [2]. This excitation signal must not be much noised, because with increasing in noise level, the variance of estimated parameters is also increased [3].

Hammerstein model is identified by TS model. The identification of TS model consists of two steps. In the first step, the identification of premise part of the TS model is carried out. By GK algorithm, a classification of the input-output space is performed. For obtained regions, membership functions are determined. For simplicity it is taken that all membership functions are Gaussian functions.

In the second step, parameters of the local ARX models are determined. During identification, it is necessary to remove the influence of piecewise polynomial disturbance. The rejection of piecewise polynomial disturbance is performed with introduction of an appropriate filter by which the input and output signal is filtered [4]. Parameter estimation is carried out by Kaczmarz algorithm [5] based on filtered observations.

The methodology presented in this paper is demonstrated through simulation.

II. TAKAGI-SUGENO MODEL

TS model is a combination of logical and mathematical model. Logical rules are considered of fuzzy premise, and the consequent is a mathematical function. The general form of TS model [6]:

$$R_i : IF\, \mathbf{x}_k\ IS\ A_i\left(\mathbf{x}_k\right) THEN\ y_{k+1}^i = f_i\left(\boldsymbol{\varphi}_k\right) \quad i = 1,\ldots,c, \tag{3}$$

where c is the number of rules, $\mathbf{x}_k \in R^n$ is the vector of premise variables, $\boldsymbol{\varphi}_k \in R^m$ is the vector of the consequent variables and A_i denotes the premise fuzzy set for the ith input. The consequent function could be any mathematical function.

In this paper, as consequent functions are used local ARX models given by relation

$$\hat{y}_{k+1}^i = \sum_{j=1}^{n_a} a_{ij} y_{k-j+1} + \sum_{j=1}^{n_b} b_{ij} u_{k-j+1}, \tag{4}$$

where a_{ij} and b_{ij} are parameters of the local ARX model.

The output of TS model is given by expression

$$\hat{y}_{k+1} = \frac{\sum_{i=1}^{c} \beta_i\left(\mathbf{x}_k\right) \hat{y}_{k+1}^i}{\sum_{i=1}^{c} \beta_i\left(\mathbf{x}_k\right)}, \tag{5}$$

where $\beta_i\left(\mathbf{x}_k\right)$ denotes the degree of fulfilment of the ith rule.

Introducing the normalised rule degree of fulfilment given by following expression

$$h_i\left(\mathbf{x}_k\right) = \frac{\beta_i\left(\mathbf{x}_k\right)}{\sum_{i=1}^{c} \beta_i\left(\mathbf{x}_k\right)}, \tag{6}$$

equation (5) becomes

$$\hat{y}_{k+1} = \sum_{i=1}^{c} h_i\left(\mathbf{x}_k\right) \hat{y}_{k+1}^i. \tag{7}$$

From relation (3) and (5) is evident that TS model approximates nonlinear system with finite collection of linear systems.

III. ESTIMATION OF TAKAGI-SUGENO MODEL

A. Estimation of Premise Membership Functions

A fundamental property of measurements for defining cluster is similarity. Therefore, it is necessary to determine the appropriate metrics. Consider an n+1-dimensional vector of measurements $\mathbf{z}_k = \left[\mathbf{x}_k, y_{k+1}\right]^T$. Set of N measurements is denoted with $\mathbf{Z} = \left\{\mathbf{z}_k \mid k = 1,2,\ldots,N\right\}$, $\mathbf{Z} \in R^{(n+1)\times N}$.

In pattern recognition terminology [7], the columns of matrix $\mathbf{Z}$ are called patters and rows are called features or attributes. The matrix $\mathbf{Z}$ is called the matrix of patterns or data.

Mahalanobis distance is introduced as a measure of similarity

$$D_M\left(\mathbf{z}_i, \mathbf{z}_j\right) = \left(\mathbf{z}_i - \mathbf{z}_j\right)^T \mathbf{F}^{-1} \left(\mathbf{z}_i - \mathbf{z}_j\right), \tag{8}$$

where $\mathbf{F}$ is a covariance matrix. Using distance (8) Gustafson-Kessel algorithm is obtained [8].

Gustasfon-Kessel clustering algorithm is an iterative optimisation algorithm for the minimisation of objective function

$$J\left(\mathbf{Z}; \mathbf{V}, \mathbf{U}, \left\{\mathbf{F}_i\right\}\right) = \sum_{i=1}^{c} \sum_{k=1}^{N} \left(\mu_{i,k}\right)^m D_M^2\left(\mathbf{z}_k, \mathbf{v}_i\right), \tag{9}$$

with constrains

$$\begin{aligned} &\mu_{i,k} \in [0,1]; \quad 1 \le i \le c; 1 \le k \le N, \\ &\sum_{i=1}^{c} \mu_{i,k} = 1; \quad k = 1,\ldots,N, \end{aligned} \tag{10}$$

where m is the weighting exponent. The outputs of the algorithm are: fuzzy partition matrix $\mathbf{U} = \left[\mu_{i,k}\right]^{c\times N}$, prototype (centres of clusters) matrix $V \in R^{(n+1)\times c}$ and c-tuple of covariance matrices $\mathbf{F}_i \in R^{(n+1)\times(n+1)}$.

Once the partition matrix is determined, it is necessary to determine the parameters of membership functions. Gaussian membership functions are defined with two parameters centre and variance of membership function. The centres of fuzzy clusters obtained executing GK algorithm are the centres of Gaussian membership functions. It is still necessary to determine the variances of the Gaussian membership functions using following expression [9]:

$$\sigma_{i,j}^2 = \frac{\sum_{k=1}^{N} \mu_{i,k} \left(x_{j,k} - v_{j,k}\right)^2}{\sum_{k=1}^{N} \mu_{i,k}}. \tag{11}$$

The degree of fulfilment of the ith rule is given by expression

$$\beta_i\left(\mathbf{x}_k\right) = w_i \mathbf{A}_i\left(\mathbf{x}_k\right) = w_i \prod_{j=1}^{n} A_{i,j}\left(x_{kj}\right), \tag{12}$$

where w_i are weights of rule and $A_{i,j}(x)$ denotes membership function for the ith rule and jth premise variable. These membership functions are defined by following expression

$$A_{i,j}\left(x_j\right) = \exp\left(-\frac{1}{2} \frac{\left(x_j - v_{i,j}\right)^2}{\sigma_{i,j}^2}\right). \tag{13}$$

Once the centres and the variances are determined (12) can be rewritten as follows

$$\begin{aligned} \beta_i\left(\mathbf{x}_k\right) &= w_i \mathbf{A}_i\left(\mathbf{x}_k\right) = \\ &= w_i \exp\left(-\frac{1}{2}\left(\mathbf{x}_k - \mathbf{v}_j^x\right)^T \left(\mathbf{F}_i^{xx}\right)^{-1} \left(\mathbf{x}_k - \mathbf{v}_j^x\right)\right), \end{aligned} \tag{14}$$

where $\mathbf{F}_i^{xx}$ denotes the diagonal matrix in which the main diagonal entries are variances of Gaussian functions for the ith rule

$$\mathbf{F}_i^{xx} = \begin{bmatrix} \sigma_{1,i}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,i}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{n,i}^2 \end{bmatrix}. \quad (15)$$

Finally, when premise membership functions are determined parameters of local models can be estimated.

B. Parameter Estimation of Consequent Functions

The output of the system on Fig. 1 is given by following expression

$$y_{k+1} = \left(1 - A\left(q^{-1}\right)\right) y_{k+1} + B\left(q^{-1}\right) f\left(v_k\right) + D\left(q^{-1}\right) d_k. \quad (16)$$

The Hammerstein model in the system can be replaced with identified TS model, and the output is defined with relation

$$\begin{aligned} y_{k+1} = \sum_{i=1}^{c} h_i\left(x_k\right)\left[\left(1 - A_i\left(q^{-1}\right)\right) y_{k+1} + B_i\left(q^{-1}\right) v_k\right] + \\ + D\left(q^{-1}\right) d_k. \end{aligned} \quad (17)$$

In order to eliminate the influence of the piecewise polynomial disturbance d_k it is necessary to introduce a filter [4] of the form

$$T\left(q^{-1}\right) = \left(1 - q^{-1}\right)^{\bar{m}+1}, \quad (18)$$

which satisfies the following relation

$$T\left(q^{-1}\right) d_k = 0. \quad (19)$$

Multiplying the expression (17) with the polynomial $T\left(q^{-1}\right)$ is obtained

$$\begin{aligned} T\left(q^{-1}\right) y_{k+1} = \sum_{i=1}^{c} h_i\left(x_k\right)\left[\left(1 - A_i\left(q^{-1}\right)\right) T\left(q^{-1}\right) y_{k+1} + \right. \\ \left. + B_i\left(q^{-1}\right) T\left(q^{-1}\right) v_k\right] + D\left(q^{-1}\right) T\left(q^{-1}\right) d_k. \end{aligned} \quad (20)$$

Introducing following notes for the filtered signals

$$\tilde{y}_{k+1} = T\left(q^{-1}\right) y_{k+1}, \quad (21)$$

$$\tilde{v}_k = T\left(q^{-1}\right) v_k, \quad (22)$$

and based on (19) the expression (20) gets the following form

$$\tilde{y}_{k+1} = \sum_{i=1}^{c} h_i\left(z_k\right)\left[\left(1 - A_i\left(q^{-1}\right)\right) \tilde{y}_{k+1} + B_i\left(q^{-1}\right) \tilde{v}_k\right]. \quad (23)$$

This expression can be written in vector form

$$\hat{\tilde{y}}_{k+1} = \hat{\boldsymbol{\theta}}_{k-1}^T \boldsymbol{\Phi}_k, \quad (24)$$

where parameter vector is

$$\hat{\boldsymbol{\theta}}_k = \left[a_{11}, \ldots, a_{1n_a}, b_{11}, \ldots, b_{1n_b}, a_{c1}, \ldots, a_{cn_a}, b_{c1}, \ldots, b_{cn_b}\right]^T. \quad (25)$$

The vector $\boldsymbol{\Phi}_k$ is defined as follows

$$\boldsymbol{\Phi}_k = \left[h_1\left(z_k\right) \boldsymbol{\varphi}_k^T, , h_c\left(z_k\right) \boldsymbol{\varphi}_k^T\right]^T = \mathbf{h}\left(z_k\right) \otimes \boldsymbol{\varphi}_k, \quad (26)$$

where $\otimes$ denotes Kroneker product, the vector of observations is defined as

$$\boldsymbol{\varphi}_k = \left[-y_k, \ldots, -y_{k-n_a}, v_{k-1}, \ldots, v_{k-n_b}\right]^T \quad (27)$$

and the vector of normalised degrees of fulfilment of the fuzzy rules has following form

$$\mathbf{h}\left(z_k\right) = \left[h_1\left(z_k\right), \ldots, h_c\left(z_k\right)\right]^T. \quad (28)$$

Now Kaczmarz algorithm can be applied to estimate the vector of parameters $\boldsymbol{\theta}$

$$\begin{aligned} \hat{\boldsymbol{\theta}}_k &= \hat{\boldsymbol{\theta}}_k + \frac{\boldsymbol{\Phi}_k}{\boldsymbol{\Phi}_k^T \boldsymbol{\Phi}_k}\left(y_k - \boldsymbol{\Phi}_k{}^T \hat{\boldsymbol{\theta}}_{k-1}\right) \\ \hat{\boldsymbol{\theta}}(0) &= \mathbf{0} \end{aligned} \quad (29)$$

IV. SIMULATION

In simulation, it is used Hammerstein model, which nonlinear part is given by following cubic function

$$v(k) = v(k) + 0.65 v^2(k) + 0.35 v^3(k), \quad (30)$$

and linear part is

$$\frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)} = \frac{0.8 q^{-1} + 0.5 q^{-2}}{1 - 0.8 q^{-1} + 0.6 q^{-2} - 0.4 q^{-3}}. \quad (31)$$

The system is excited with multi-sinusoidal input, whose amplitude is bounded in the range $[-1.5, 1.5]$. Gaussian noise $N(0, 0.1)$ is added to multi-sinusoidal signal.

Disturbance used in simulation is polynomial piecewise signal, defined by random quadratic functions. Disturbance dynamic is represented by following relation

$$\frac{D\left(q^{-1}\right)}{A\left(q^{-1}\right)} = \frac{1 - 0.4 q^{-1}}{1 - 0.8 q^{-1} + 0.6 q^{-2} - 0.4 q^{-3}}. \quad (32)$$

The form of input, disturbance and output signal used for identification of TS model are shown on Fig. 3.

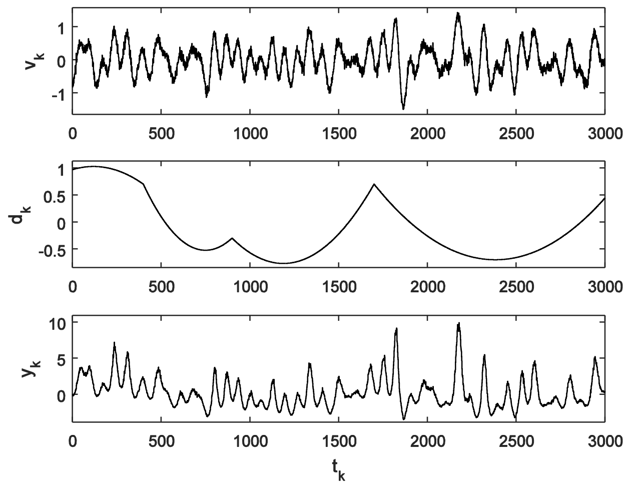

Fig. 3. Input, disturbance and output of the system used for identification of TS model.

The TS model have one premise variable v_k. The optimal number of cluster is obtained using performance measures

[10]. The minimum of fuzzy hypervolumens and maximum of partition densities is for $c = 2$.

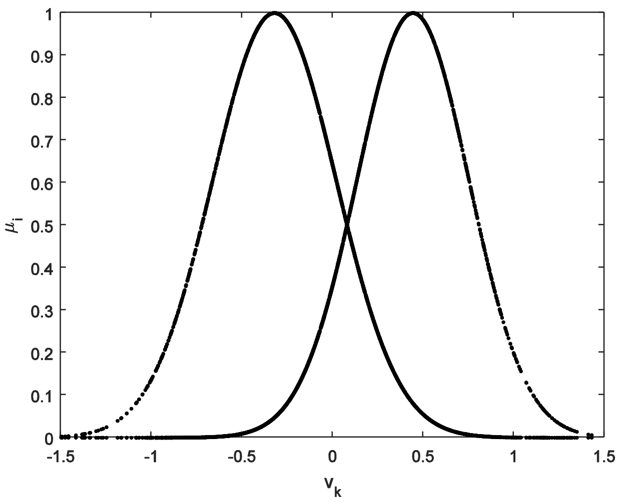

Fig. 4. Estimated membership functions.

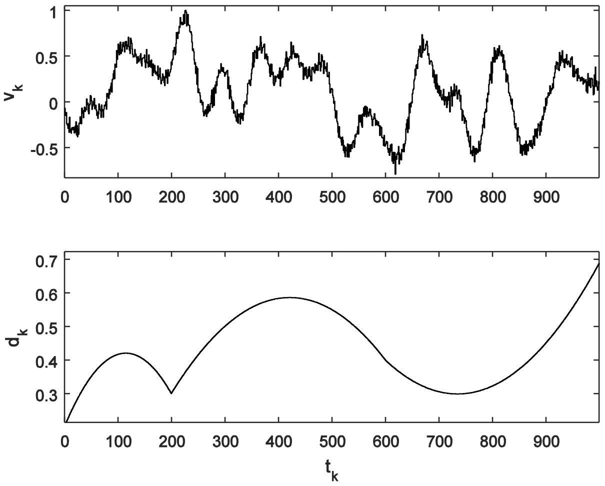

Fig. 5. Input and disturbance used for validation of TS model.

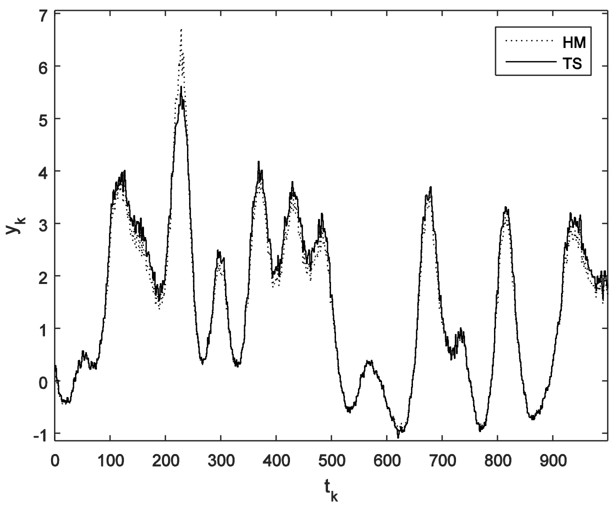

Fig. 6. Outputs of the system with Hammerstein model and the system with identified TS model.

In Fig. 4 are shown the estimated Gaussian membership function for the premise variable. New input signal and piecewise polynomial disturbance are generated for the verification of TS model, Fig. 5. In Fig. 6 outputs of system with Hammerstein and TS model are compered.

V. CONCLUSION

In the paper, the method of identification of TS model has been discussed in the case when piecewise polynomial disturbance acts on the system. The state space is divided into fuzzy regions using Gustafson-Kessel algorithm. For each fuzzy region the parameters of Gaussian membership functions are estimated. Thereafter, the parameters of the local ARX models are estimated. The effect of this disturbance is eliminated using the appropriate filter. The simulation has shown good agreement between the output of the system and the case when the part of the system is replaced by TS model.

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