

QUANTITATIVE METHODS FOR DETERMINING PREMIUM RATES IN TRANSPORTATION INSURANCE UNDER UNCERTAINTY

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Abstract: *In decision-making processes, including insurance decision-making processes, decisions are usually made in the circumstances of uncertainty or absence of information or knowledge of a particular problem, so it is necessary to conduct different assessments and find different solutions for the problem. Actuaries and underwriters are faced with many problems in the process of quantifying risk in conditions of uncertainty, where classical techniques are difficult to apply because of a large number of unknown parameters. Given the specificity of transportation insurance, the technique of fuzzy mathematics and the possibility theory can be used to a great extent in various transportation insurance lines, including the segment of determining the premium rates.*

Keywords: *uncertainty, fuzzy measures, c-credibility, transportation insurance, premium rates.*

1. INTRODUCTION

There is a large number of transportation insurance lines, and the most prevailing is marine cargo insurance. This insurance means insurance of the various tangible interests of many participants involved in the processes of the physical distribution of goods. This implies providing security to the insured who is exposed to various risks that can occur during the process of transportation, warehousing and manipulation of the goods, by providing financial compensation for any damage that the insured would sustain if the insured transport perils occur. The subject matter of insurance is the goods as well as the financial interests of the policyholders who are exposed to the transportation risks (Tomašić, 1987, Seltmann, 2004, Dunt, 2009, Thomas, 2009, Pavić, 2012).

Actuarial science is multidisciplinary, a combination of science and practical applications, so the subjectivity of an actuary in the parameters evaluation is significant, but also the uncertainty of occurrences in certain tasks, for example in the quantification

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of the risk parameters needed to determine the premium rates. The practice has shown that the difficulties exist in the correct setting of the problem, as well as in the subjective interpretation of the results of the experiment.

Also, in insurance practice there is often high level of complexity of the observed problems, when the experience based on human expertise is taken into account, instead of precise calculation (Gajović and Paunović, 2018). In addition to actuarial science, a very important segment in the insurance business is underwriting, especially in transportation insurance. Underwriting involves the process of analysis, selection and classification of insurance applications, assessment of the potential clients' exposure to certain risks, as well as determination of the conditions and rates of insurance cover (DeWit, 1982; 2009). Kunreuther et al. (1995) state that actuaries usually calculate the level of insurance risk and the value of insurance coverage on the basis of available statistical data, while underwriters and insurance managers usually make final decisions on accepting risks, the elements in the insurance offer and final insurance premium.

In order that the underwriters decisions should be as correct and reliable as possible, underwriters must also have, in addition to their knowledge of the insurance lines they are specialized in, a broad knowledge of actuarial theory and practice. A managerial decision of the underwriters in non-life and transportation insurance is usually made on the basis of information obtained from the client, policy holder (eg business entities, brokers, freight forwarders, carriers, agents, ports, etc.), a competent person from insurance company, outsourced personnel responsible for inspecting the subject of the insurance, institutions competent for assessing the creditworthiness of the client, etc.

In many cases, transportation insurance is contracted on the basis of the underwriter decisions. There are many reasons for this (Gajović, 2015):

- a large number of homogeneous groups of insured subject matter,
- numerous elements and sub-elements of risks,
- different modalities of insurance contracts that cover different risks or groups of risks,
- changes in transportation risks in space and time,
- limitations of coverage by insured or insurance company, etc.

In decision-making processes, including insurance decision-making processes, decisions are usually made in *the circumstances* of uncertainty or absence of information or knowledge of a particular problem, so it is necessary to make different assessments and to find different solutions to the problem. Uncertainty implies phenomena whose outcome cannot be correctly predicted in advance. The uncertainty theory is a part of mathematics which, among other things, deals with modeling the belief degree. The possibility theory is a part of the theory of uncertainty for the processing of incomplete information and it is based on a pair of dual measures, the possibility measure and the necessity measure. Taking into account the specificity of transport insurance, the techniques of mathematics and the possibility theory can find significant application in various transport insurance lines, including the segment of determining premium rates. Fuzzy mathematics and fuzzy system has proved to be applicable in the risk assessment (Paunović et al., 2018; Gajović et al., 2018) and other segments of theory and practice of insurance such as classification, risk underwriting (Kerkez and Gajović, 2016), liabilities projections, future and present values (Cummins, 1987), calculating of insurance

premiums (Young, 1996, 2004 Hosler, 1992), asset allocation, cash flows, investments (Lemaire, 1992, Ostaszewski, 1993) etc.

The classical methods of determining insurance premiums in transport insurance and the problems faced by actuaries and underwriters when determining premium rates will be set out in the following parts. Then the fuzzy measures are defined, as well as two measures of credibility in the fuzzy environment. The c-credibility measure is a new measure that represents the aggregate value of the necessity and possibility measures. Particularly discussed are the fuzzy logic and the fuzzy relations as well as the method of assigning value to relations. The above-mentioned quantification methods of transport rates are presented through the relevant examples.

2. CLASSICAL METHODS OF DETERMINING TRANSPORT INSURANCE PREMIUM

The processes of inflow and outflow of financial assets in insurance are stochastic and can be very complex. The basic approach to the insurance risk calculation is the assumption that total loss costs from all insured risks over a given period of time arises due to a stochastic process to which the insurer does not have any influence or its impact is limited. Since frequency and amount of claims represent a stochastic category, it is necessary to determine their individual distributions of probability and distribution parameters. They are most often performed on the basis of historical data used to determine the theoretical distribution characteristics, or certain approximations and assumptions are used, in the absence of a representative statistical sample. The insurance premium is a parameter that is directly related to the risk of insurance coverage.

Risk premium formula is

$$E(LC) = f_k \cdot s, \quad (1)$$

where

f_k the number of claims per exposure unit, and

s is severity (average loss per claim).

The risk premium is further adjusted by the influencing factors related to the usual actuarial techniques, limits of insurance, deductible, as well as the available data on the claims and incidents in the previous period. The further premium risk correction relates to the specificity of the insured, such as industry classification of the insured, the characteristics of the goods in the transport, the specificity of the transportation, the transport destination, etc.

There are many specific features of transport insurance, which differentiate it from other non-life insurance lines. Transport insurance lines are often complex and require a multi-disciplinary knowledge, coverage is often adapted to the specific needs of the clients etc. Unlike most other non-life insurance lines, transport insurance is often based on international law, international conventions (CMR, CMNI, CMR, TIR, ATA, FIATA, ADR, RID and others) and business ethics (INCOTERMS, for ports, shipping, packaging, security, etc.), international terms and conditions and insurance clauses, etc. Numerous authors point to the specificity of transport insurance and the complexity of the process of determining premium rates, especially in the insurance of goods in transport. Given

the fact that insurance coverage conditions are most often tailored to the special needs and requirements of the insurance policyholder, contracts in cargo insurance often cover individual, specific risks not covered by the standard conditions or insurance clauses (Gajović, 2015).

The distribution of total claims is determined on the basis of the premium adequacy in relation to the projected expectation on losses with risk loading coefficient for the purpose of calculating the required actuarial premium (Biener, 2011). Given that the goal of any insurer is to provide the required insurance coverage with the necessary actuarial premium, the basic problem is how to ensure assessment of sufficient actuarial premium, to cover losses incurred in the future period, and to provide financial resources to settle all future obligations of insurers and actual profit from business. In addition to the total costs of the future claims, the insurer has significant additional costs incurred by the insurer organization (eg employee compensation costs, management costs, external costs, reinsurance costs, etc.). The most common indicators for determining accuracy of actuarial and underwriters methods and assumptions are as follows: loss ratio, loss adjustment expense ratio - LAE ratio, UW expense ratio, operating expense ratio - OER, combined ratio and others. The most common methods for calculating the premium of various nonlife insurance lines based on statistics are pure premium method and loss ratio method. In the world practice, there are different modalities of those methods, such as experience rate, exposure rating, manual rating, schedule rating, retrospective rating (Neuhaus, 2004, Werner and Modlin, 2010). In case of transportation insurance lines, the judgment rate making methods are used in practice as a result of the lack of a representative statistical sample that causes low credibility. The premium is usually estimated on the basis of intuition, experience or on the basis of available, non-representative data, and it depends on the characteristics of the insurance coverage, different specificities and limitations. In practice, expert assessment of risk insurance coverage is carried out by the underwriters, who are most often specialized for certain types of insurance and have extensive knowledge and experience of risk management in relation to the subject of insurance and insurance coverage. The established premium or premium rate determined by the statistical or expert-based underwriters methods can be adjusted, depending on the number of influences on the level of risk of insurance coverage, various costs, modalities and specificities of the various insurance contracts, etc.

There are many corrective factors that often need to be considered and included in the various nonlife and transportation insurance policies when deciding on the final premium rate for specific insurance coverage, such as (Gajović, 2015):

- **Reinsurance.** Reinsurance is the most common instrument for transfer of a part of risk over the own insurer capacity. The risks that the insurer keeps for himself is called retention. Determining the reinsurance premium depends on the type of reinsurance contract and the specificity of the insurance coverage and implies the application of complex mathematical and actuarial methods and models (Daykin et al., 1995; Schmitter, 2004). Reinsurance premium has direct influence on the gross premium.
- **Deductible.** Deductible is the amount that is deducted, in case of occurrence, as a certain percentage or amount, depending on the contract. By contracting a deductible, the insurer transfers a part of the risk on the insured resulting in reduced

risk coverage of the insurer and in the same time stimulating insured to carry out activities in the function of minimizing damage. The type and amount of the deductible in contract significantly affect the amount of the insurance premium (Buchanan i Priest, 2004).

- **Type of insurance contract.** According to their duration, transportation insurance contracts may be voyage or time contracts, and that has direct influence on the insurance premium amount. Insurance of goods in transport can be contracted through individual or general contracts (Dunt, 2009; Pavić 2012, Tomašić, 1987). The most common types of general cargo insurance contracts are the Open cover policy, Floating policy, Fronting insurance contracts, etc.
- **Utility theory.** Utility theory is widely used in the field of risk management and insurance due to the fact that the behavior of decision-makers on the amount of risk and its acceptability is described in accordance with the individual character of money. From the point of view of the insurance institution, the insured person often decides to pay a premium that is higher than the expected level of loss due to the utility theory and the principle of decreasing marginal utility of money (Brown and Gottlieb, 2001, Daykin et al., 1995).
- **Market factor.** The competition is very heavy on the international and domestic insurance market affecting the reduction of insurance premiums, flexibility of the insurer related to the insurance contracting, expanding the range of insurance products, flexibility regarding modalities of payment of insurance premiums and others. A number of authors deal with the influence of the market on the price of insurance and market process modeling in insurance.
- **Corporate objectives.** The goal of each insurance company is to make the greatest possible profit from the insurance business. Insurance companies may have different policies for risk acceptance or strategies that imply readiness, appetite, or tolerance regarding risk acceptance.
- **Characteristics of insured.** Insurer monitors the available information about the policy holder and the insured, their creditworthiness, reputation, business reputation, basic activities, possession of appropriate business standards, examines their own and experience of the other insurers, and on that basis makes decision on the modality and the price of insurance coverage, possible limitations and, in certain cases, rejecting to provide insurance.

In some cases, these factors may be of key importance in risk assessment and may, in some situations, affect the decision of the insurer on not accepting insurance risk.

In practice, different principles of calculating premium are applied, including the area of transport insurance. The principles are actuarial rules on the basis of which insurance companies determine the premium or premium rates for the assumed risks. In the literature, there are several methods that actuaries use in defining premium principles such as Ad Hoc Method, Characterization Method, Economical Method, etc. The most common principles for determining the premium are Net Premium Principle, Expected Value Premium Principle, Variance Premium Principle, Standard Deviation Premium Principle, Equivalent Utility Principle, Exponential Premium Principle, Esscher Premium Principle (Young, 2004, 1999, Kaas and et al., 2009) and others.

Credibility can be used to calculate the price and premium rates, to determine the future premium rates based on the experience, claims provision etc. In the classical credibility theory there are two basic methods, Limited Fluctuation methods, and the Method of the greatest accuracy. When applying credibility, it is necessary to consider the characteristics of both the experience of the company and the relevant experience (experience similar to the company experience).

The basic credibility formula is

$$\text{Estimation} = Z \cdot [\text{Observation}] + (1 - Z) \cdot [\text{Other information}], \quad (2)$$

where $0 < Z < 1$, Z credibility assigned to observation, and $1-Z$ is referred to as the complement of credibility.

Credibility implies a linear assessment of the real expectations made as a result of a compromise between observation and the previous hypothesis. Because of the uncertainty of phenomena certain difficulties exist in the real application.

Example 1. In a large population of truck drivers, the average driver makes a claim every four years, i.e. the annual claim frequency is 0,25 per year. The result of a performed random selection is a driver who had three claims over the last five years with a claim frequency of 0,55 per year. An estimate of the expected future frequency for this driver is needed. If there is no additional information about the driver, except that he belongs to the observed population, the frequency would have been 0,25. However, we know that the claim frequency for the observed driver was 0,55. There is a correlation between the previous claim frequency and the future claim frequency, but it is not perfect correlation. Due to the randomness of accidents, even the good drivers with low expected claim frequencies can be involved in some accident. On the other hand, drivers with poor driving skills can file no claims for a couple of years. According to the general formula of credibility (2), the expected value of future drivers incident frequency is

$$Z \cdot 0,55 + (1 - Z) \cdot 0,25.$$

The necessary number of data for credibility is called the standard for full credibility. Thus some regulators prescribe the required sample size necessary for full credibility for certain lines of insurance. For example, in Canada for liability insurance, it is 3.246, while for collision insurance that number is 1.082.

There are two methods for determining the weight in the above formula.

According to the limited fluctuation method, Z is determined as follows:

$$Z = \min \left(\sqrt{\frac{N_{comp}}{N_{fcr}}}, 1 \right) \quad (3)$$

N_{comp} - the number of insureds in the most recent historical period of the company

$N_{forecast}$ - the size of the sample needed for full credibility

According to the second method, models of a greatest accuracy credibility, formula for Z is:

$$Z = \frac{N_{comp}}{N_{comp} + K}, \quad K = \frac{\tau}{a} \quad (4)$$

where K is the ration of the expected value of process variance and variance of hypothetical meanings.

By definition, each individual can choose any value for these parameters. The fact is that in practice the most commonly used 95% level of reliability and 5% for marginal error, does not provide scientific foundation. The actuary might want to use the other values for these parameters. As a result, for the same insurance, an actuary may consider, for example, 400 for the sample size that produces complete credibility for the experiential data. The other actuary may take into account any number of samples less than 1100 that is not sufficient the complete credibility (Klugman and Rhodes, 2009; Kerkez and Ralević, 2018).

3. THE POSSIBILITY THEORY

In the 1980s, many authors pointed to the strong link between Dempster Shafer's theory, the theory of probability and the possibility theory based on the fuzzy measures. The fuzzy measure theory is the generalization of the classical theory of the measure. This generalization is obtained by replacing the additive axiom of classical measure with the weaker axioms of monotonicity and continuity. The development of the fuzzy measure theory is motivated by the fact that the property of additive in some applications is too restrictive and, consequently, unrealistic. Some authors use the term monotone measures instead fuzzy measures. Thus, the fuzzy measure according to Sugeno is obtained by replaced additive properties with the conditions of monotonicity (in relation with inclusion) and continuity, but it was subsequently determined that the continuity is also restrictive, and it is replaced by the condition of semi continuity.

The name of possibility theory was introduced in Zadeh (1978), inspired by Gaines and Kohouta (1975). To a large extent, it can be compared with the probability theory, as it is based on a set function. Possibility and necessity measures, as monotone and semi continuous measures, are increasingly applied in real problems.

Definition 1. Let X be nonempty set and Σ σ -algebra on X . Fuzzy measure (*fuzzy measure in the narrow sense*) m on σ - algebra Σ is map $m : \Sigma \rightarrow [0, \infty]$ with properties

$$i) \quad m(\emptyset) = 0, \tag{5}$$

$$(ii) \quad (\forall R, S \in \Sigma) R \subset S \Rightarrow m(R) \leq m(S). \tag{6}$$

The belief measure Bel and plausibility measure Pl are special forms of monotonous measures. The measure connection with preconceived impression is called the belief measure. Measure related to information that is possible or acceptable (plausible) is called a measure of plausibility (Dubois and Prade, 1980).

Definition 2. Necessity function is a fuzzy measure on (X, Σ) if

$$nec\left(\bigcap_{i \in I} R_i\right) = \inf_{i \in I} nec(R_i), \tag{7}$$

for any family of subsets $\{R_i \mid i \in I\}$ in Σ such that $\bigcap_{i \in I} R_i \in \Sigma$, where I is an arbitrary index set.

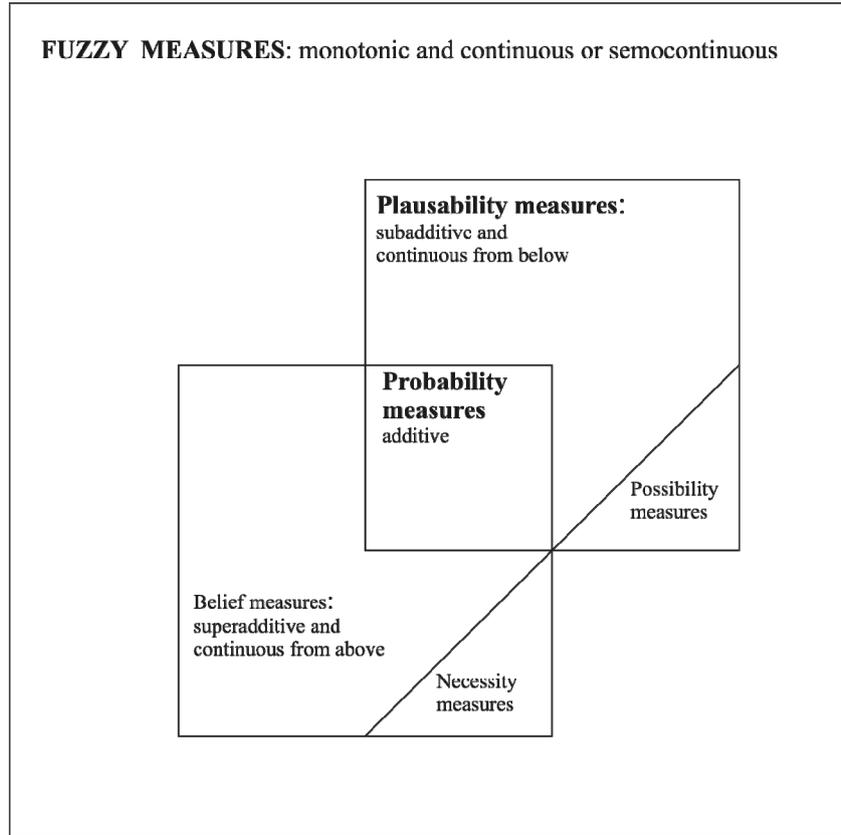


Figure 1. Relationship between the fuzzy measure classes

Definition 3. Possibility measure is a function

$$pos\left(\bigcup_{i \in I} R_i\right) = \sup_{i \in I} pos(R_i), \tag{8}$$

for any family of subsets $\{R_i \mid i \in I\}$ in Σ such that $\bigcup_{i \in I} R_i \in \Sigma$, where I is an arbitrary index set.

Relationship between the fuzzy measure classes is shown in Figure 1.

An alternative version of the credibility theory in the fuzzy environment, as a branch of mathematics for studying behavior of fuzzy phenomena, was formulated by Liu (2007). The credibility measure (Liu and Liu, 2002) is presented as a no additive measure with self-duality property, as an average of possibility and necessity measures:

$$Cr(R) = \frac{1}{2}(pos(R) + nec(R)), \tag{9}$$

where R is the set in possibility space $(X, P(X), pos)$.

In the classical theory of credibility, the main task is to find weight of measures. However, the weight here is predetermined and it is 0,5. For this very reason it is necessary to generalize the credibility measure, and that is achieved with c-credibility measure given by Ralević and Paunović (2018).

The generalization of this measure in fuzzy environment is achieved through the aggregation function. Aggregation of information takes a significant place in many knowledge-based systems, where aggregation of data or values is needed. In general, it can be said that by aggregation simultaneously, different parts of information from different sources are used, in order to make a conclusion or a decision.

Aggregation operation (Klir and Yuan, 1995) on n fuzzy sets ($n > 2$) is defined by a function $h: [0,1]^n \rightarrow [0,1]$.

When applied to fuzzy sets A_1, A_2, \dots, A_n defined on X , function h produces an aggregate fuzzy set A by operating on the membership grades of these sets for each $x \in X$.

Thus,

$$A(x) = h(A_1(x), A_2(x), \dots, A_n(x)) \text{ for each } x \in X. \quad (10)$$

The c -credibility in the fuzzy environment in relation to the aggregation function h is defined in the following way

$$cr_h(R) = h(pos(R), nec(R)) \quad (11)$$

If aggregation function is a weighted arithmetic mean

$$h(x, y) = \lambda \cdot x + (1 - \lambda) \cdot y, \lambda \in [0,1] \quad (12)$$

(which is not symmetrical in the general case), then c -credibility in the fuzzy environment is

$$cr_\lambda(R) = \lambda \cdot pos(R) + (1 - \lambda) \cdot nec(R), \quad (13)$$

In insurance, it is often necessary to determine the appropriate premium rates for the future, i.e. to adjust the past premium to the expected value in the future period. Thus, an actuary must also take in the calculation changes in future costs due to inflation impacts, regulatory changes, technology advances, etc. Normally, the premium also includes profit and contingencies. In addition, the premium must be adjusted to reflect all premium trends, which is achieved through the so-called trend factors. In the following example, the method of determining future premium rates based on lost cost method (Brown and Gottlieb, 2001) is presented, and calculations are performed using classical actuarial technique and c -credibility.

Data on premium rates and claims by regions are given in Table 1. We need to calculate indicated rate change.

Table 1. Insurance company data by regions.

Region	Current Base rates	Earned premium at current rates	Incurred losses	Claim Count	Units of exposures
A	153	830000	330000	866	5424,8366
B	63	965000	525000	1100	15317,4603
C	102	550000	290000	320	5392,15686

For the lost cost method, we need to calculate the Units of exposures using the formula (1), ie. as ratio of earned premium rate at current rates and current base rates. Current

differential is the ratio of current base rates to the base rate of the region with highest incurred losses. Indicated differential is calculated by the same process by comparing the data for the loss cost.

Adopted differential (table 2) is then aggregation function $(Z \cdot d_i^n + (1-Z) \cdot d_c^n)^{\frac{1}{n}}$ for the different values of the parameter k .

Table 2. Result for loss cost model

Region	Current diff.	Loss Cost	Indicated diff.	Z	Adopted			
					k=1	k=2	k=3	k=4
A	2,428571	60,83	1,775	0,895	1,844	1,855	1,867	1,882
B	1,000000	34,27	1,000	1,000	1,000	1,000	1,000	1,000
C	1,619048	53,78	1,569	0,544	1,592	1,592	1,592	1,592

4. FUZZY RELATIONS AND FUZZY LOGIC

The concept of the relation can be generalized to provide a different degree or strength of link between the elements. The fuzzy relationship is a fuzzy set defined on the space $X = X_1 \times X_2 \times \dots \times X_n$, where $(x_1, x_2, \dots, x_n) \in X$ can have different degrees of membership in the relation.

Definition 4. Let X_1, X_2, \dots, X_n are arbitrary sets and $X_1 \times X_2 \times \dots \times X_n$ is Cartesian product. Fuzzy set F defined on $X^n (n > 1)$ is the n -ary fuzzy relation on X .

For fuzzy relation F on $X_1 \times X_2 \times \dots \times X_n$, value $F(x_1, x_2, \dots, x_n)$ is the degree, ie. the level of connectivity of elements x_1, x_2, \dots, x_n , in sequence, by criteria F . The fuzzy relationship is identify with its membership function, so it is indicated in the literature by $\mu_F(x_1, x_2, \dots, x_n)$ or $F(x_1, x_2, \dots, x_n)$.

For fuzzy relation F , the conditions of reflexivity, symmetry, antisymmetry and transitivity are fulfilled. It is simply checked that for the reflexive and transitive fuzzy relation F is $F \circ F = F$. If the fuzzy relation on the set X is reflexive, symmetric, and transitive, then it is fuzzy equivalence on X .

Phase equivalence F on a set is called a fuzzy equality if for all $x, y \in X$, holds $F(x, y) = 1$ then $x = y$.

For binary fuzzy relation F on $X \times Y$ we have:

The first projection of a fuzzy relation F is a fuzzy set on X is defined by

$$(\text{dom } F)(x) = \sup_{y \in Y} F(x, y) \tag{14}$$

The second projection of a fuzzy relation F is a fuzzy set on Y is defined by

$$(\text{rang } F)(y) = \sup_{x \in X} F(x, y) \tag{15}$$

$(\text{dom } F)(x)$ is the strongest relation of $x \in X$ with any member of set Y , and $(\text{rang } F)(y)$ is the strongest relation of $y \in Y$ with any member of set X .

Let $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_m\}$ are finite sets. Membership matrix of fuzzy relation is a matrix $A_F = [a_{ij}]_{n \times m}$ with elements $a_{ij} = F(x_i, y_j) \in [0, 1]$.

Let F is binary fuzzy relation represented with membership matrix $A_F = [a_{ij}]_{n \times m}$, then for any $x_i \in X, y_j \in Y$ $(\text{dom } F)(x) = \max_{j \in \{1, \dots, m\}} a_{ij}$ is the maximal element in the i -th row, and $(\text{rang } F)(y) = \max_{i \in \{1, \dots, n\}} a_{ij}$ is the maximal element in the j -th column.

Composition of fuzzy relations can be defined in several ways. The following is the definition of the standard composition resulting from the operation min and max .

Let F_1 and F_2 are fuzzy relations respectively on $X \times Y$ and $Y \times Z$.

Definition 5. Standard composition of relation F_1 and relation F_2 is relation ρ on $X \times Z$, defined by

$$\rho(x, z) = (F_1 \circ F_2)(x, z) = \sup_{y \in Y} \min \{F_1(x, y), F_2(y, z)\}. \quad (16)$$

Fuzzy relation ρ can be represented by its membership matrix.

Let $A_{F_1} = [a_{ij}]_{n \times k}$ and $A_{F_2} = [b_{ij}]_{k \times m}$ membership matrices of fuzzy relations F_1 and F_2 . Membership matrix of the fuzzy relation $\rho = F_1 \circ F_2$ is $A_\rho = [c_{ij}]_{n \times m}$, where $c_{ij} = \max_{s \in \{1, \dots, k\}} \min \{a_{is}, b_{sj}\}$ for all $i \in \{1, 2, \dots, n\}$ and $j \in \{1, 2, \dots, m\}$.

Composition of fuzzy relation can also be defined through other operations. There are several ways of assigning values to relations. Methods that attempt to determine a kind of similar form or structure of data through different metrics are called similarity methods (see Zadeh, 1973 or Dubois and Prade, 1980). There are dozens of these methods (see Ross, 1995), and through the example we will introduce the max-min method.

Example 2. An insurance company has formed a team of experts to determine certain classes of damage on goods in road transport, for a particular type of goods, in the observed geographical area. The following table shows the results of the expert analysis by the levels of damage: without damage, moderate damage and major damage.

Table 3. Results of the expert analysis

Region	x_1	x_2	x_3	x_4	x_5
without damage	0,4	0,2	0,6	0,1	0,2
moderate damage	0,7	0,4	0,6	0,2	0,4
major damage	0,2	0,5	0,2	0,2	0

For max-min method formula for elements of fuzzy relation is

$$a_{ij} = \frac{\sum_{k=1}^m \min(x_{ik}, x_{jk})}{\sum_{k=1}^m \max(x_{ik}, x_{jk})}, \quad i, j = 1, 2, \dots, n. \quad (17)$$

Using the previous formula, we obtain a symmetric matrix that shows the similarity of damage by areas.

$$A_p = \begin{bmatrix} 1 & & & & & \\ 0,500 & 1 & & & & \\ 0,800 & 0,471 & 1 & & & \\ 0,385 & 0,455 & 0,357 & 1 & & \\ 0,462 & 0,545 & 0,429 & 0,375 & 1 & \end{bmatrix}$$

Further, through the equivalence relation, the areas can be classified into certain categories of similar properties, as an indicator of damages (claims), as well as a correction factor for future premium rates.

Introducing labels that indicate more precise descriptions and gradual transitions is the method of quantification of the uncertainty by fuzzy logic (Dubois and Prade, 1987). Fuzzy logic implies the membership degree of an element to a particular set. It can belong to different sets, but with a certain degree. There is a gradual transition between the sets, and they can overlap in a certain degree. Fuzzy logic is the generalization of classical logic in the way in which the fuzzy sets are generalization of classical sets.

Truth value $T(P)$ assigned to some proposition P in fuzzy logic can take some arbitrary value from interval $[0,1]$, ie. $T : P \rightarrow (0,1)$. This actually means that in fuzzy logic exist a degree of truth (truthfulness) of some statement. This degree of truth is derived from the membership functions of fuzzy set on which the statement defined.

Suppose that proposition P is assigned to fuzzy set A , then the truth value of the proposition is $T(P)$, $T(P) = \mu_A(x)$, where $0 \leq \mu_A(x) \leq 1$, i.e. the degree of truth for $P : x \in A$ is equal to the membership degree of x to the fuzzy set A .

Let proposition P is defined on the fuzzy set A and proposition Q defined on set B , then we have:

i) $T(\bar{P}) = 1 - T(P)$ negation

ii) $P \vee Q : x$ is A or B ,

$$T(P \vee Q) = \max(T(P), T(Q)) \text{ disjunction}$$

iii) $P \wedge Q : x$ is A and B ,

$$T(P \wedge Q) = \min(T(P), T(Q)) \text{ conjunction}$$

iv) $P \rightarrow Q : x$ is A , then x is B ,

$$T(P \rightarrow Q) = T(\bar{P} \vee Q) = \max(T(\bar{P}), T(Q)) \text{ Zadeh's implication.}$$

The often used implications is Lukasiewicz's implication

$$T(P \rightarrow Q) = \min(1 - T(P) + T(Q), 1) \quad (18)$$

Implication can be presented in the form of rules

$P \rightarrow Q$: if x is equal to A , then y is equal to B , and that equivalent with fuzzy relation

$\rho = (A \times B) \cup (\bar{A} \times Y)$ same as in classical logic.

Then membership function of ρ is represented by the formula

$$\mu_\rho(x, y) = \max[(\mu_A(x) \wedge \mu_B(y)), (1 - \mu_A(x))]. \quad (19)$$

Example 3. Suppose we evaluate the possibility of changing a certain transport insurance product in order to estimate the expected effects of its placement on the market. We use two metrics to make a decision about validity for product change. The units of measurement are the "uniqueness of the adjusted product", marked on the scale of uniqueness, $X = \{1, 2, 3, 4\}$ and "market size" for the product, denoted on the scaled market size, $Y = \{1, 2, 3, 4, 5, 6\}$. On the both scales, the smallest numbers are assigned to "the greatest uniqueness" and "the largest market". Let's say that the product is rated "middle uniqueness", marked by the set E and the mean market size, the set F . We want to determine the implications of such a result, ie. IF E THEN F .

Let:

$$E = 0,5/2 + 0,2/3 + 0,3/4,$$

$$F = 1/1 + 0,5/2 + 1/3 + 0,7/4 + 0,3/5.$$

and row vector $Y = 1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6$,

Using the formula (6) and formula for Cartesian product

$$\mu_\rho(x, y) = \mu_{E \times F}(x, y) = \min(\mu_E(x, y), \mu_F(x, y))$$

we have

$$E \times F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0,5 & 0,4 & 0,1 & 0,5 & 0,1 & 0 \\ 1 & 0,4 & 0,1 & 0,6 & 0,1 & 0 \\ 0,2 & 0,2 & 0,1 & 0,2 & 0,1 & 0 \end{bmatrix}, \bar{E} \times Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0,8 & 0,8 & 0,8 & 0,8 & 0,8 & 0,8 \end{bmatrix}$$

and further $\rho = \max(E \times F, \bar{E} \times Y)$, i.e.

$$\rho = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 \\ 1 & 0,4 & 0,1 & 0,6 & 0,1 & 0 \\ 0,8 & 0,8 & 0,8 & 0,8 & 0,8 & 0,8 \end{bmatrix}$$

5. CONCLUSION

Availability of information of numerous risk parameters in transportation insurance, collected from the often limited past period, affects the reliability of risk estimates and accuracy of determining premium rates. In certain cases, in this line of insurance, it is possible to access statistical data on realized risks, but the data are most often not representative. For this reason, in order to analyze, evaluate, and control insurance risk, the economic, legal, engineering and technological knowledge, intuition and experience of various experts are often used in practice. Managerial decisions in practice imply subjectivity, uncertainty and the possibility of errors. The main causes of uncertainty and errors are the complexity and ambiguity of the problem, often the lack of accurate information, inaccurate records or data analyses and the subjective perception of the problem.

Classical actuarial methods can hardly be applied in terms of uncertainty, ambiguity, low credibility of data, and other limitations. However, combination of the various quantitative methods that are regularly applied in finance and insurance and techniques and principles based on fuzzy mathematics, can be significant support for actuaries and underwriters in decision-making. C-credibility measure in the fuzzy environment was introduced as a regular measure in relation to the aggregate value of possibility and necessity measures. Premium rates in nonlife or transportation insurance are often only calculated at the average level with the approximate values so application of the c-credibility measure can give a more accurate premium rates closer to the characteristics of each insured. Also, the concept of relations with generalization is presented for a different degree or strength of connection between the observed elements.

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REFERENCES

- [1] Brown, R., Gottlieb, L. (2001). Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance. 2nd Edition. ACTEX Publications Inc.
- [2] Buchanan, R., Priest, C. (2004). Deductible, *Encyclopedia of Actuarial Science*, Vol. 1, Editors-in-Chief, Teugels J., Sundt B., 445-447.
- [3] Cummins, D., Derrig, R. A. (1997). Fuzzy Financial Pricing of Property-Liability Insurance, *North American Actuarial Journal*, 1 (4) 21-40.
- [4] Daykin, C. D., Pentikäinen, T., Pesonen, E. (1995). *Practical Risk Theory for Actuaries*, Chapman and Hall, London.
- [5] De Wit, G. W. (1982). Underwriting and Uncertainty, *Insurance: Mathematics and Economics*, 277-285.
- [6] Dubois, D., Prade, H. (1987). Necessity measure and resolution principle. *IEEE Transactions on Man Cybernet*, 17, 474-478.
- [7] Dubois, D., Prade, H. (1980). *Fuzzy Sets and Systems: Theory and Applications*. Academic Press, Boston.

- [8] Dunt, J. (2009). Marine Cargo Insurance, Informa, Lloyd's Shipping Law Library.
- [9] Gaines, B.R., Kohout, L. (1975). Possible automata. Proc. Int. Symp. Multiple-Valued logics, Bloomington, IN 183-196.
- [10] Gajović, V. (2015). Modeliranje rizika u logističkim procesima sa primenom u transportnom osiguranju, Doktorska disertacija, Univerzitet u Beogradu, Saobraćajni fakultet.
- [11] Gajović, V., Paunović, M. (2018). Applying fuzzy mathematics to risk assessment in insurance industry. Tokovi osiguranja, 1, 23-38.
- [12] Hosler, V. R. (1992). The application of fuzzy sets to group health underwriting, ARCH 2, 1-63.
- [13] Kaas, R., Goovaerts, M., Dhaene, M.J., Denuit, M. (2009). Modern Actuarial Risk Theory, Springer.
- [14] Kerkez, M., Gajović, V. (2016). Underwriting risk assessment in marine cargo insurance, Risk management in the financial services sector, (Kočović, J. et al. eds), University of Belgrade, Faculty of Economics Publishing Centre, 381-399.
- [15] Kerkez, M., Ralević, N. M. (2018). Uncertainty analysis and risk modeling in insurance, Insurance in the post-crisis era, (Kočović, J., Jovanović Gavrilović, B., Boričić, B., Radović Marković, M., eds), University of Belgrade, Faculty of Economics Publishing Centre, 2018, Chapter 18, 309-326.
- [16] Klir, G. J., Yuan, B. (1995). Fuzzy sets and fuzzy logic, theory and applications, Prentice Hall, New Jersey.
- [17] Klugman S., Rhodes, T., Purushotham, M., Gill, S., MIB Solutions. (2009). Credibility Theory Practices. Society of Actuaries.
- [18] Kunreuther, H., Meszaros J., Hogarth R., Spranca, M. (1995). Ambiguity and underwriter decision processes, *Journal of Economic Behavior & Organization* 26, 337-352
- [19] Lemaire, J. (1990). Fuzzy Insurance, ASTIN Bulletin, 20 (1), 33-55.
- [20] Liu, B. (2007). Uncertainty Theory, (2nd ed.), Springer-Verlag, Berlin.
- [21] Liu, B., Liu, Y. K. (2002) Expected Value of Fuzzy Variable and Fuzzy Expected Value Models, IEEE Transactions on Fuzzy Systems, 10, 445_450.
- [22] Neuhaus, W. (2004). Experience-rating, *Encyclopedia of Actuarial Science*, Vol. 2, Editors-in-Chief, Teugels J., Sundt B., 639-646.
- [23] Ostaszewski, K. (1993) Fuzzy sets Methods in Actuarial Science, Schaumburg IL, Society of Actuaries.
- [24] Paunović, M., Ralević, N. M., Gajović, V., Mladenović-Vojinović, B., Milutinović, O. (2018). Two-Stage Fuzzy Logic Model for Cloud Service Supplier Selection and Evaluation, Mathematical Problems in Engineering, MATH PROBL ENG, ISSN: 1024-123X, Hindawi Publishing Corporation.
- [25] Paunović, M., Ralević, N., Milutinović, O., Vojinović, Ž., Mladenović-Vojinović, B. (2019). Integrated Fuzzy System and Multi-Expression Programming Techniques for Supplier Selection, Tehnički vjesnik - Technical Gazette, 26 (1), 122-127.
- [26] Pavić, D. (2012). Pomorsko osiguranje, pravo i praksa s osnovama kopnenoga i zračnog transportnog osiguranja, Književni krug Split.
- [27] Ralevic, N. M., & Paunovic, M. (2019). c– Credibility Measure. Filomat, 33(9), 2571-2582.
- [28] Ross, T. (1995). Fuzzy Logic and Engineering Applications, 1st ed., McGraw-Hill, New York.

- [29] Schmitter, H. (2004). Reinsurance Pricing, Encyclopedia of Actuarial Science, Vol. 3, Editors-in-Chief, Teugels J., Sundt B., 1404-1410.
- [30] Seltmann, A. (2004). Marine Insurance, Encyclopedia of Actuarial Science, Vol. 2, Editors-in-Chief, Teugels J., Sundt B., 1057-1063.
- [31] Thomas, D. R. (2009). The Modern Law of Marine Insurance. Informa, London.
- [32] Tomašić, V. (1987). Transportno osiguranje, Savremena administracija, Beograd.
- [33] Werner, G., Modlin, C. (2010). Basic Ratemaking. Casualty Actuarial Society. EMB, Version 4.
- [34] Young, V. R. (2004). Premium principles, Encyclopedia of Actuarial Science, Vol. 3, Editors-in-Chief, Teugels J., Sundt B., 1323-1331.
- [35] Young, V. R. (1996). Insurance Rate Changing: a Fuzzy Logic Approach, Journal of Risk and Insurance, 63 (3), 461-484.
- [36] Zadeh, L. (1973). Outline of a new approach to the analysis of complex systems and decision processes, IEEE Trans. Syst. Man, Cybern., 3, 28-44.
- [37] Zadeh, L. (1978). Fuzzy Sets as a basis for a theory of Possibility, Fuzzy Sets and Systems 1, 3-28.